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RF POWER ALLOCATION FOR A UNIFIED TELEMETRY SYSTEM

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-64-104

JUNE 1964

L. L. Stine

Prepared for

DIRECTORATE OF AEROSPACE INSTRUMENTATION

ELECTRONIC SYSTEMS DIVISION

AIR FORCE SYSTEMS COMMAND

UNITED STATES AIR FORCE

L. G. Hanscom Field, Bedford, Massachusetts

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ABSTRACT

The document describes the power spectrum of a RF carrier phase modulated by several frequency-multiplexed signals. It is proved, in text and in illustration, that an improvement in power allocation is possible by modulating one signal on a separate RF carrier.

REVIEW AND APPROVAL

Publication of this technical documentary report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

Roy D. Ragsdale

ROY D. RAGSDALE
Colonel, USAF
Director, Aerospace Instrumentation

RF POWER ALLOCATION FOR A UNIFIED TELEMETRY SYSTEM

SECTION I

INTRODUCTION

Integrated telemetry systems, such as the Space-Ground Link Subsystem (SGLS), send information from a space vehicle to the ground by frequency multiplexing the various data functions and phase-modulating an RF carrier. The functions include FM and PAM/FM according to IRIG standards, PCM/PM with varying bit rates up to one megabit per second, FM voice, and ranging tones.

In order to maximize power in the various data channels, a large percentage of available power is wasted in intermodulation products. This document will show under what conditions significant increases in power budgeting efficiency may be attained by modulating a high-power consuming function, such as one megabit PCM on a separate RF carrier.

A power-budgeting optimization procedure is described. A budget when all SGLS functions are modulated on one carrier is compared with a budget when the PCM function is modulated on a separate carrier.

SECTION II

THE BASEBAND SPECTRUM

A unified telemetry system accommodates each function by phase or frequency-modulating a subcarrier which, in turn, phase modulates the RF carrier. Each function is assigned a unique subcarrier. The composite signal shows a series of frequency slots, each occupied by a particular function. The resulting normalized signal is

$$s(t) = \cos [w_c t + m_1 \cos w_1 t + m_2 \cos w_2 t + \ldots\ldots m_k \cos w_k t] \ , \qquad (1)$$

where

w_c is the RF carrier radian frequency,

w_i is the subcarrier radian frequency for the i^{th} function, and

m_i is the modulation index of subcarrier i.

Giacolleto [1] has derived an equivalent expression for the signal:

$$s(t) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \cdots\cdots \sum_{n_k=-\infty}^{\infty} \prod_{i=1}^{k} J_{n_i}(m_i) \cos \left[w_c + \sum_{i=1}^{k} n_i w_i \right] t \ , \qquad (2)$$

where

$J_n(m_i)$ is the n^{th} order Bessel function with argument m_i.

The carrier component of the signal is

$$c(t) = \prod_{i=1}^{k} J_o(m_i) \cos w_c t \ . \qquad (3)$$

The component at each subcarrier frequency is

$$i(t) = J_1(m_i) \prod_{j=1}^{i-1} J_o(m_j) \prod_{j=i+1}^{k} J_o(m_j) \left[\cos(w_c + w_i)t - \cos(w_c - w_i)t \right]. \tag{4}$$

All other components are intermodulation products.

SECTION III

PROOF OF DOUBLE-CARRIER EFFICIENCY IMPROVEMENT

Let P_{SCT} be the total information power in k subcarriers modulating a single carrier. Let P_{DCT} be the total information power in k-1 subcarriers modulating one carrier plus the power of a completely suppressed second carrier directly modulated by the information of subcarrier k. Then

$$P_{SCT} = 2 \sum_{i=1}^{k} J_1^2(m_i) \prod_{j=1}^{i-1} J_o^2(m_j) \prod_{j=i+1}^{k} J_o^2(m_j) , \tag{5}$$

$$P_{DCT} = 2A^2 \sum_{i=1}^{k-1} J_1^2(m_i) \prod_{j=1}^{i-1} J_o^2(m_j) \prod_{j=i+1}^{k} J_o^2(m_j) + 1 - A^2 , \tag{6}$$

where

A^2 is the fraction of total power in the first carrier and its sidebands, and

$1-A^2$ is the fraction of power in the sidebands of the completely suppressed second carrier.

The ratio of two information powers is

$$\frac{P_{SCT}}{P_{DCT}} = \frac{2\left[\sum_{i=1}^{k} J_1^2(m_i) \prod_{j=1}^{i-1} J_o^2(m_j) \prod_{j=i+1}^{k} J_o^2(m_j)\right]}{2A^2\left[\sum_{i=1}^{k-1} J_1^2(m_i) \prod_{j=1}^{i-1} J_o^2(m_j) \prod_{j=i+1}^{k-1} J_o^2(m_j)\right] + 1 - A^2} , \tag{7}$$

$$\frac{P_{SCT}}{P_{DCT}} = \frac{2\left[J_o^2(m_k)\sum_{i=1}^{k-1} J_1^2(m_i)\prod_{j=1}^{i-1} J_o^2(m_j)\prod_{j=i+1}^{k-1} J_o^2(m_j) + J_1^2(m_k)\prod_{j=1}^{k-1} J_o^2(m_j)\right]}{2A^2\left[\sum_{i=1}^{k-1} J_1^2(m_i)\prod_{j=1}^{i-1} J_o^2(m_j)\prod_{j=i+1}^{k-1} J_o^2(m_j)\right] + 1 - A^2} . \tag{8}$$

Let the information power in the k^{th} subcarrier equal the power in the sidebands of the suppressed second carrier:

$$2\, J_1^2(m_k)\prod_{j=1}^{k-1} J_o^2(m_j) = 1 - A^2 \quad . \tag{9}$$

Then, for $P_{SCT}/P_{DCT} < 1$:

$$2\left[J_o^2(m_k)\sum_{i=1}^{k-1} J_1^2(m_i)\prod_{j=1}^{i-1} J_o^2(m_j)\prod_{j=i+1}^{k-1} J_o^2(m_j)\right] ,$$

$$< 2A^2\left[\sum_{i=1}^{k-1} J_1^2(m_i)\prod_{j=1}^{i-1} J_o^2(m_j)\prod_{j=i+1}^{k-1} J_o^2(m_j)\right] , \tag{10}$$

or

$$J_o^2(m_k) < A^2 \quad . \tag{11}$$

If this condition is possible, will it still maintain the equality condition of Equation (9) ?

$$J_o^2(m_k) + 2\, J_1^2(m_k) \le 1 \quad , \tag{12}$$

$$\prod_{j=1}^{k-1} J_o^2(m_j) \le 1 \quad , \tag{13}$$

$$J_o^2(m_k) + 2\, J_1^2(m_k) \prod_{j=1}^{k-1} J_o^2(m_j) \le 1 \ . \tag{14}$$

From Eq. (9),

$$2\, J_1^2(m_k) \prod_{j=1}^{k-1} J_o^2(m_j) = 1 - A^2 \ . \tag{15}$$

Substituting (15) into (14), and rearranging terms yields

$$J_o^2(m_k) \le A^2 \quad . \tag{16}$$

Thus, for condition (11), it is possible to more efficiently distribute power into the information bands by putting one function on a separate RF carrier.

Figure 1 is a plot of Eq. (9) and (11). It also shows the J_o^2 and J_1^2 functions. It is obvious that $J_o^2(m_k)$ is always less than A^2, but the discrepency is larger for large values of m_k and small values of $\prod_{j=1}^{k-1} J_o^2(m_j)$. The reason is that, as the modulation indices increase to meet large requirements for information power, more power is dispersed in intermodulation

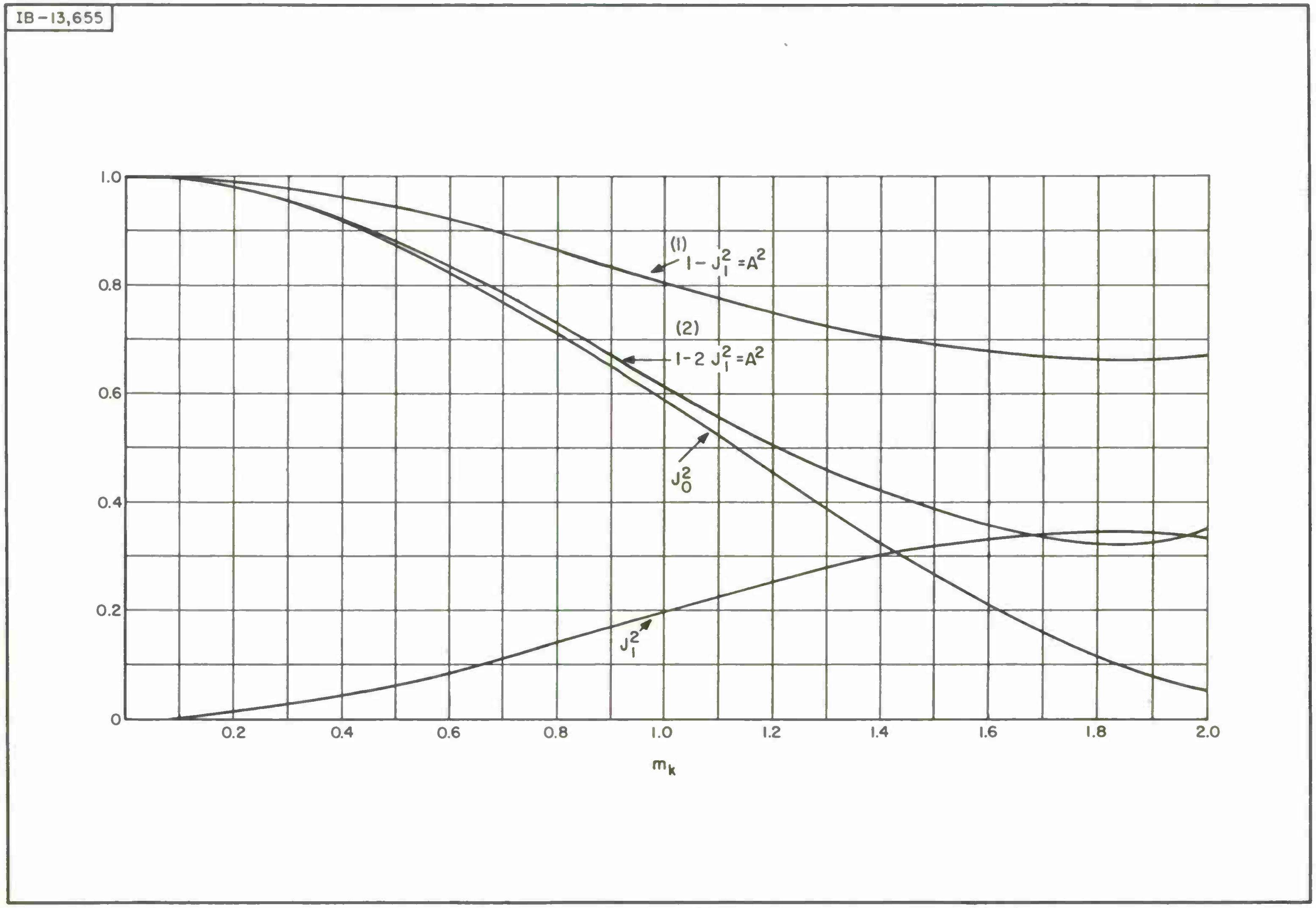

Fig. 1. A^2 vs. $J_o^2\,(m_k)$ for Eq. (9) and: (a) $\prod_{i=1}^{k-1} J_o^2(m_i) = 0.5$, and (b) $\prod_{i=1}^{k-1} J_o^2(m_i) = 1$.

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products. Thus, it is reasonable to expect that modulating one of the functions on a separate carrier, especially the function with the largest modulation index, will provide a greater improvement in efficiency.

SECTION IV

A SAMPLE POWER-BUDGETING CALCULATION WITH OPTIMIZATION

In a multiplexed communications link, each function requires a minimum received-signal level in order to detect and demodulate the corresponding information. The signal level varies according to signal bandwidth, noise power, and desired information accuracy. Table I shows the minimum signal strengths or thresholds for each of the SGLS functions and the corresponding percentage of total information power.

Table I

Information Mode Threshold Requirements

Function	Threshold Power (dbm)	Percentage (F)
PCM (10^6 bps)	-105.7	65
PAM/FM	-108.5	29.4
Voice (20 kc)	-115.5	5.85
Ranging		
500 kc	-149.5	5.9×10^{-4}
other 7 tones	-144.5	18×10^{-4}
Totals	-103.6	100

Power is apportioned between the carrier and the various subcarriers or first-order sidebands as described by Eqs. (3) and (4). The communications link will be optimum if the power in the first-order sidebands is divided in accordance with the threshold percentages and, also, so that the total power in the first-order sidebands is maximized. Thus, it is required to maximize the following:

$$\max \sum P_i \quad , \tag{17}$$

given that

$$\frac{P_1}{F_1} = \frac{P_2}{F_2} = \ldots\ldots = \frac{P_k}{F_k} \quad , \tag{18}$$

where

P_i is the first-order sideband power for the i^{th} function, and

F_i is the corresponding threshold percentage.

Table II shows the results of the optimization. The first column shows the distribution 2 watts of available power when all the functions are phase-modulated on one RF carrier. The second column shows how the same amount of power is distributed when the PCM function is directly modulated on a separate carrier.

Larry L. Stine

Larry L. Stine

Table II

A Power-Budgeting Comparison

Power Functions*	Single Carrier (Watts)	Double Carrier (Watts)
P_{PCM}	0.456 (1.00)	0.976 (A^2 = .512)
P_{PAM}	0.376 (0.92)	0.441 (1.40)
P_v	0.048 (0.36)	0.086 (0.78)
P_{500}	49×10^{-6} (0.01)	47×10^{-6} (0.02)
P_{7RT}	34×10^{-5} (0.01)	85×10^{-6} (0.01)
P_c	0.704	0.241
P_{IM}	0.418	0.256

* P_c = carrier power

P_{PAM} = PAM/FM power

P_v = analog voice power

P_{500} = 500-kc range tone power

P_{7RT} = other 7 range tone power

P_{PCM} = PCM/PM power

P_{IM} = intermodulation power

( ) = modulation index

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1. Giacolleto, L. J. "Generalized Theory of Multitone Amplitude and Frequency Modulation"; Proceedings of the IRE, Vol. 35, July, 1947, pp. 680-693.

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